

# Lecture 23: Options Markets

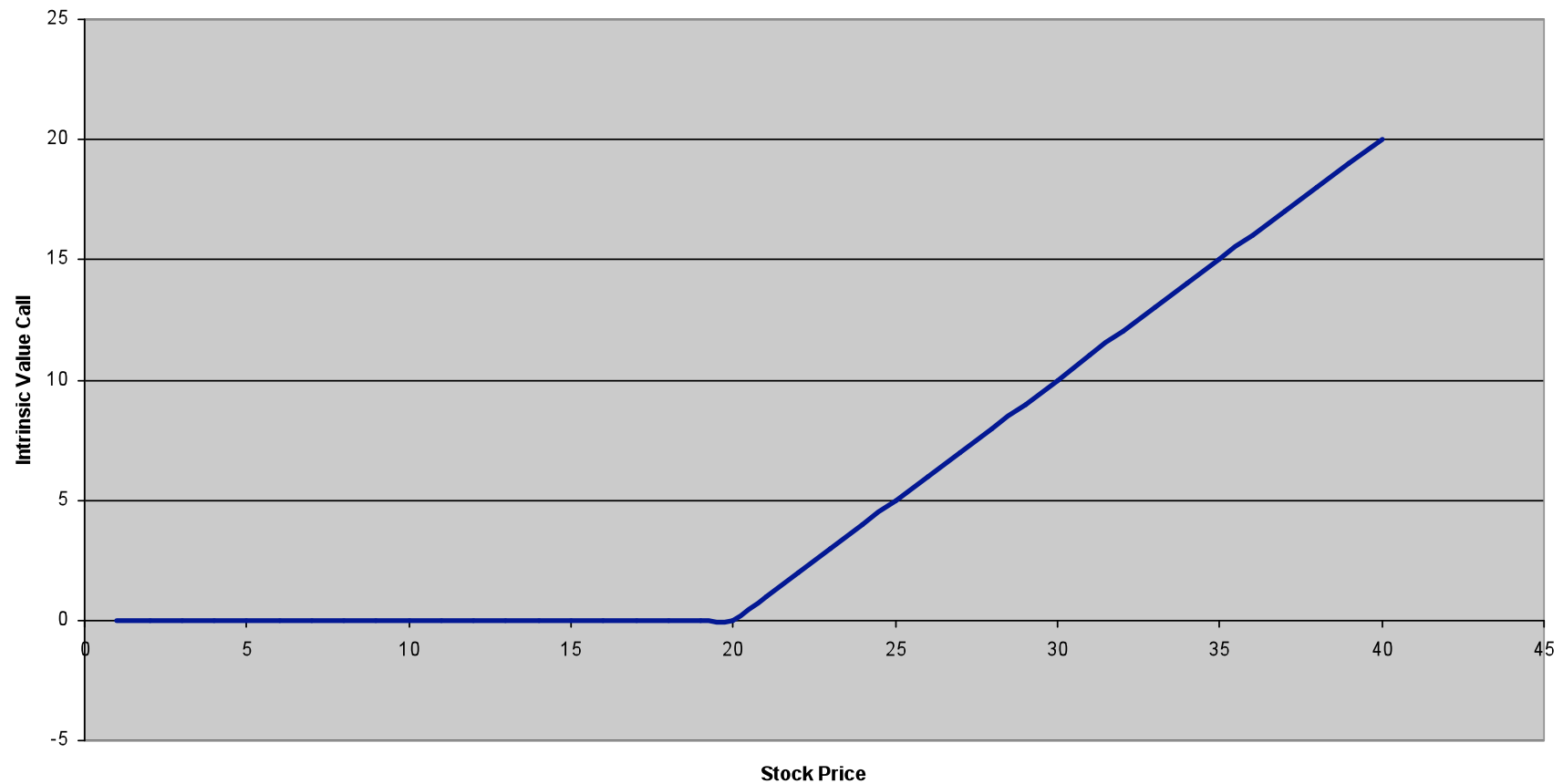
Economics 252, Spring 2008

Prof. Robert Shiller, Yale University

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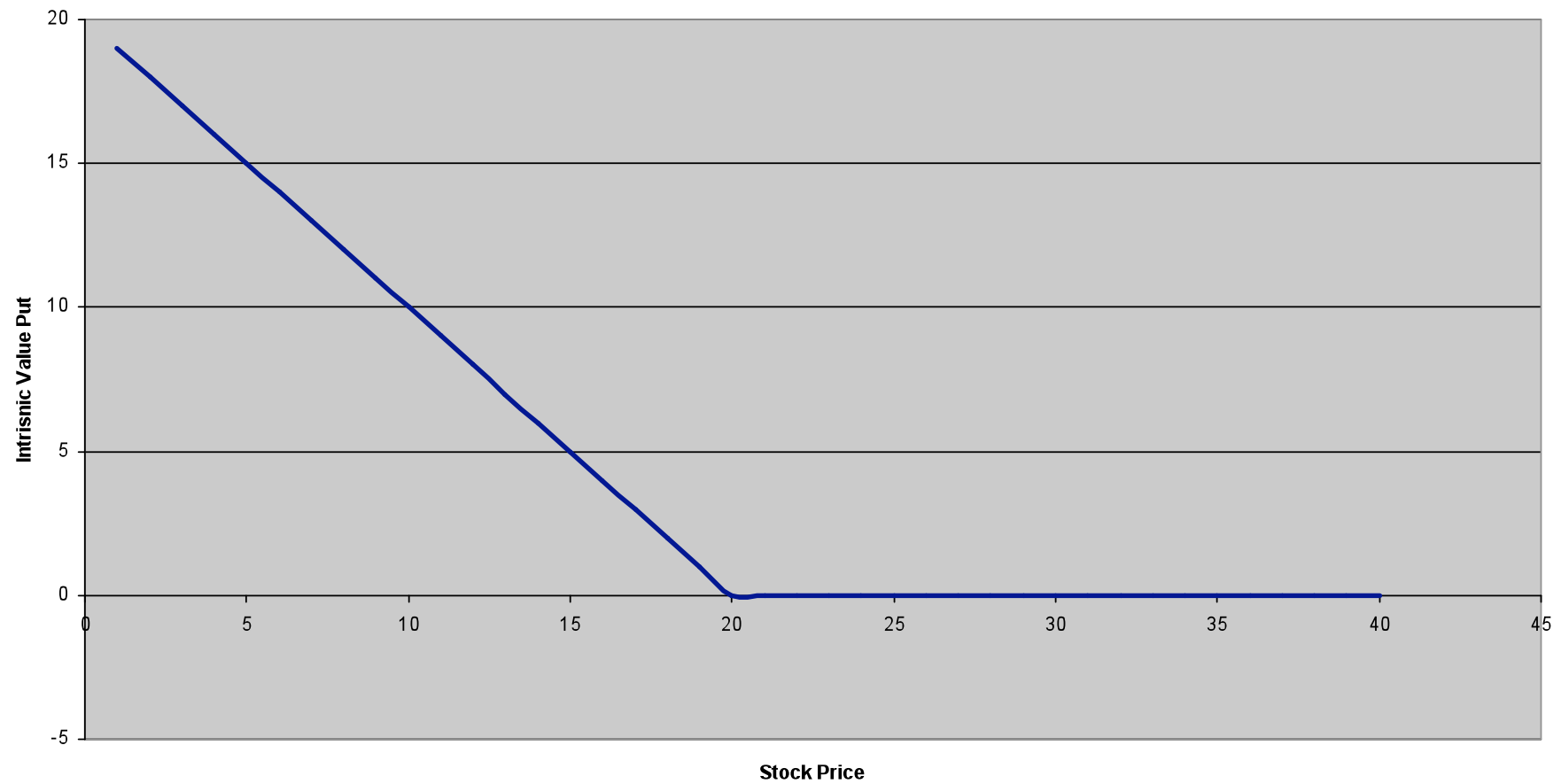
**Exercise Price = 20**



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**Exercise Price = 20**



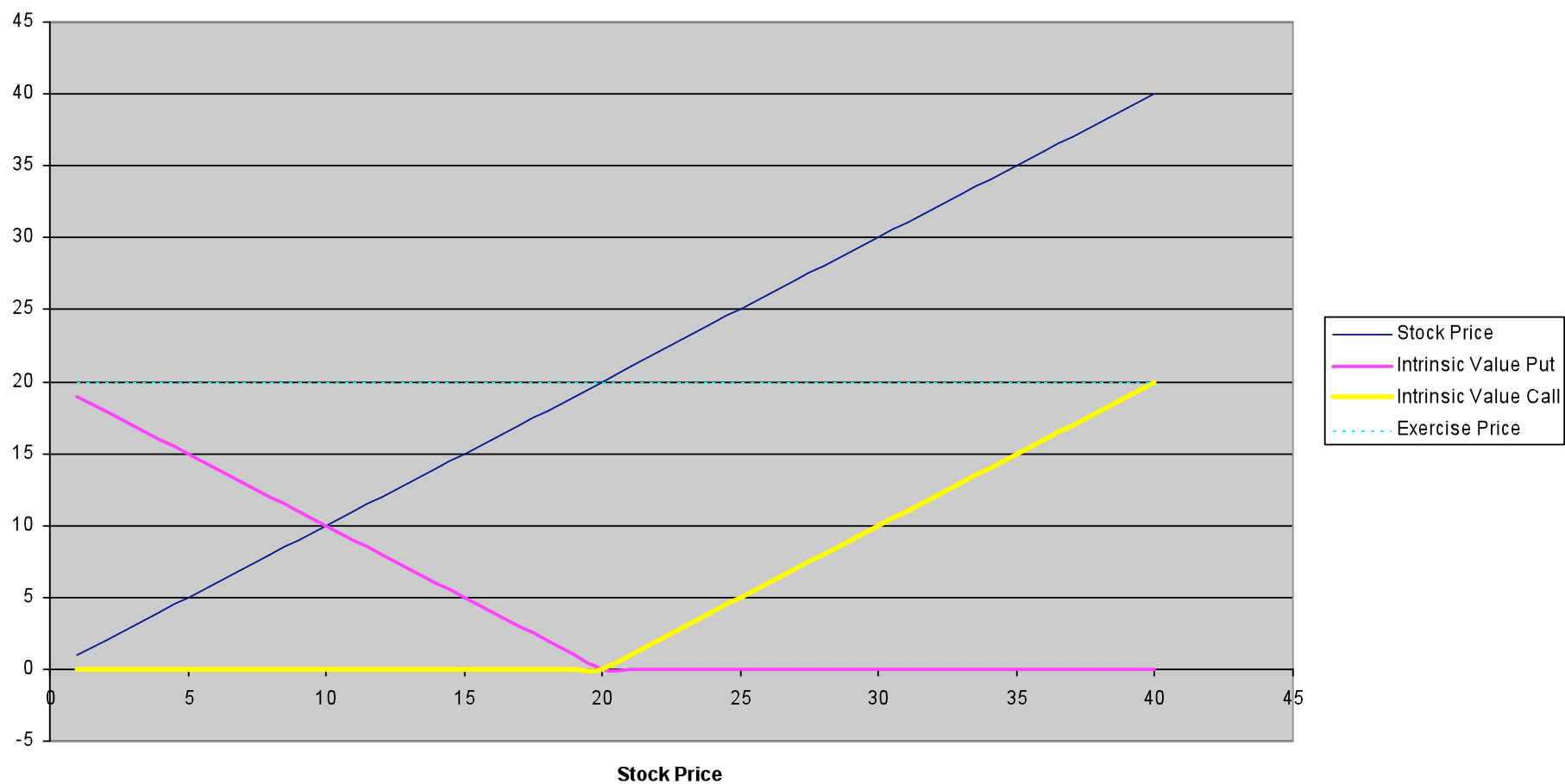
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# Put-Call Parity Relation

- Put option price – call option price = present value of strike price + present value of dividends – price of stock
- For European options, this formula must hold (up to small deviations due to transactions costs), otherwise there would be arbitrage profit opportunities

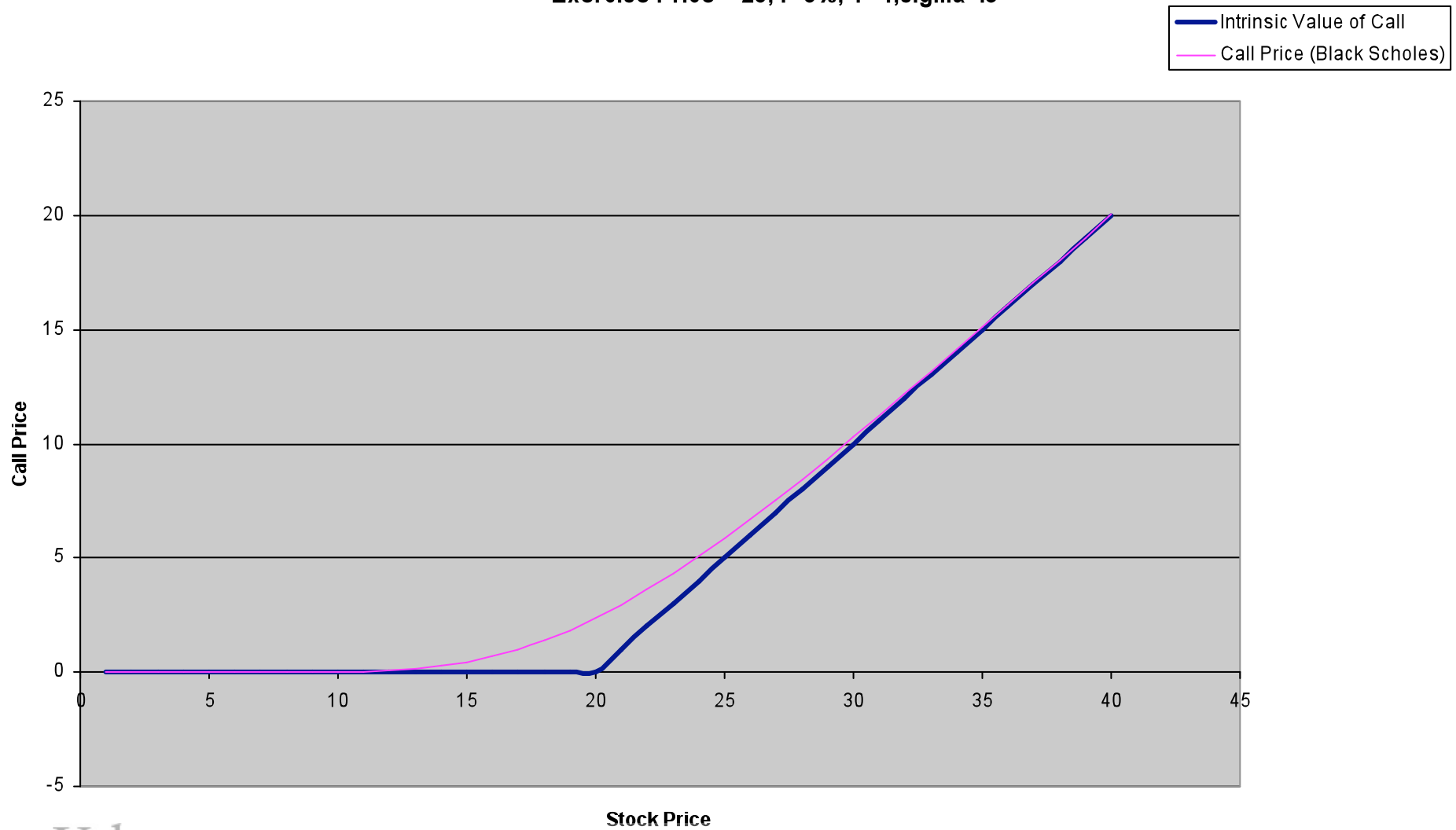
### Put Call Parity Relation Derivation



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Exercise Price = 20,  $r=5\%$ ,  $T=1$ ,  $\sigma=.3$



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# Binomial Option Pricing

- $S$  = current stock price
- $u = 1 + \text{fraction of change in stock price if price goes up}$
- $d = 1 + \text{fraction of change in stock price if price goes down}$
- $r$  = risk-free interest rate

# Binomial Option Pricing, Cont.

- $C$  = current price of call option
- $C_u$  = value of call next period if price is up
- $C_d$  = value of call next period if price is down
- $E$  = strike price of option
- $H$  = hedge ratio, number of shares purchased per call sold



# Hedging by writing calls

- Investor writes one call and buys  $H$  shares of underlying stock
- If price goes up, will be worth  $uHS - C_u$
- If price goes down, worth  $dHS - C_d$
- For what  $H$  are these two the same?

$$H = \frac{C_u - C_d}{(u - d)S}$$

# Binomial Option Pricing Formula

- One invested  $HS-C$  to achieve riskless return, hence the return must equal  $(1+r)$   
 $(HS-C)$
- $(1+r)(HS-C)=uHS-C_u=dHS-C_d$
- Subst for  $H$ , then solve for  $C$

$$C = \left(\frac{1+r-d}{u-d}\right)\left(\frac{C_u}{1+r}\right) + \left(\frac{u-1-r}{u-d}\right)\left(\frac{C_d}{1+r}\right)$$

# Black-Scholes Option Pricing

Call  $T$  the time to exercise,  $\sigma^2$  the variance of one-period price change (as fraction) and  $N(x)$  the standard cumulative normal distribution function (sigmoid curve, integral of normal bell-shaped curve) = normdist( $x, 0, 1, 1$ ) Excel ( $x$ , mean, standard\_dev, 0 for density, 1 for cum.)

# Black-Scholes Formula

$$C = SN(d_1) - EN(d_2)$$

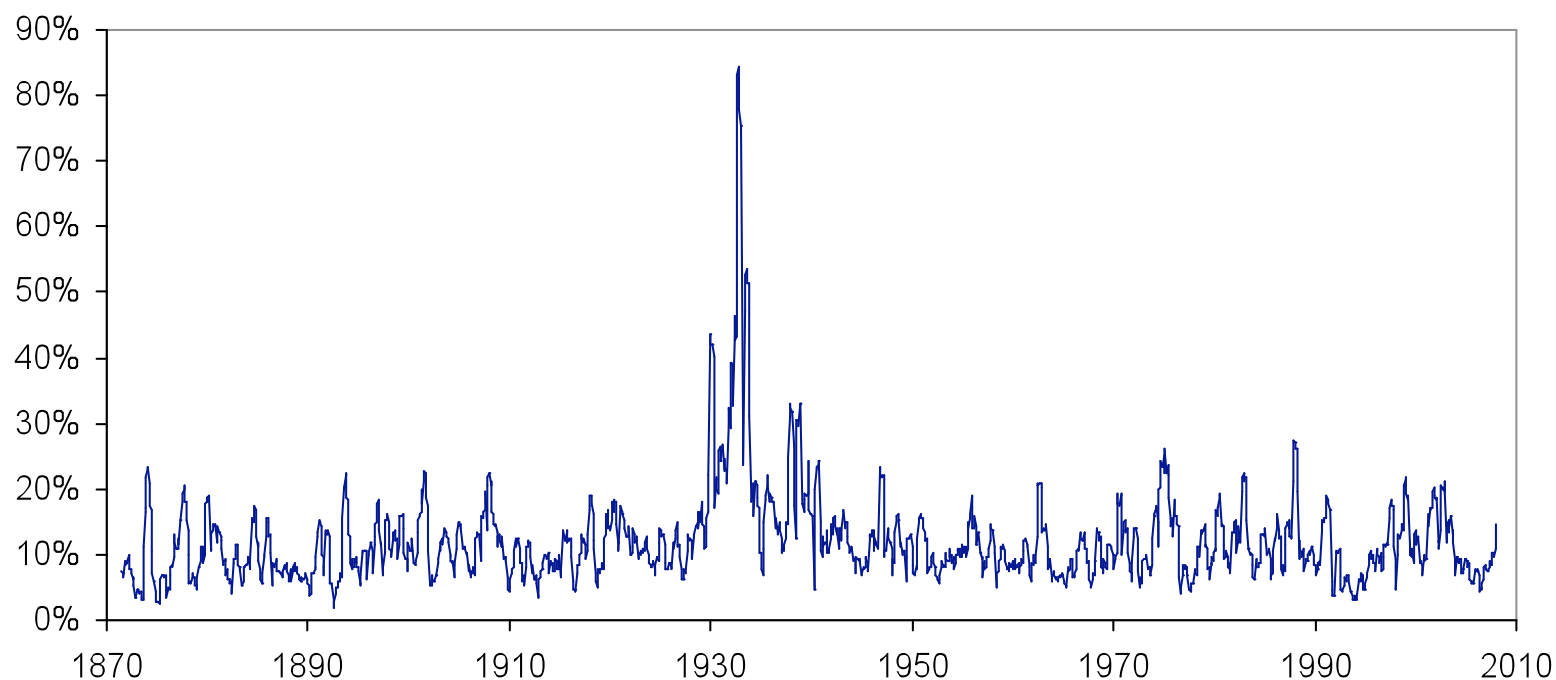
where

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + rT + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{E}\right) + rT - \sigma^2 T / 2}{\sigma \sqrt{T}}$$

# Actual S&P500 Volatility

## Monthly July 1871- April 2008



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# Implied and Actual Volatility

## Monthly Jan 1986-April 2008

