

# Answers for the Final Exam

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- This is a **closed-book** exam.
- There are **6** pages including this one.
- The exam lasts for **150** minutes (plus **30** minutes reading time).
- There are **150** total points available.
- There are **five** questions, worth **20, 15, 40, 30 and 45 points** respectively.
- Please notice that there are **FORTY-FIVE** points available in the last question.
- Please remember to attempt the easier parts of all the questions. Do not get bogged down on the hard parts: **move on!**
- Please put **each** question into a **different blue book**.
- Show your work.
- Good luck!

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## USE BLUE BOOK 1

Question 1. [20 total points] State whether each of the following claims is true or false (or can not be determined). For each, explain your answer in (at most) **one** short paragraph. Each part is worth **5** points, of which **4** points are for the explanation. Explaining an example or a counter-example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

- (a) [5 points] “William the Conqueror burned his boats because his soldiers were afraid of the dark.”

**Answer.** *False. It was a commitment strategy preventing his soldiers from being able to retreat. He burned to show the Saxons that the Normans could not retreat.*

- (b) [5 points] “Consider the strategy profile  $(s_A, s_B)$ . If player  $A$  has no strictly profitable pure-strategy deviation then she has no strictly profitable mixed-strategy deviation.

**Answer.** *True. The payoff to a mixed strategy is a weighted average of the payoffs of the pure strategies involved in the mix. So, if there were a strictly profitable mixed-strategy deviation, at least one of the pure strategies involved would have to be strictly profitable.*

- (c) [5 points] “In duel (the game with the sponges) if your probability of hitting if you shoot now plus the probability of your opponent hitting if she were to shoot next turn is greater than one, then it is a dominant strategy for you to shoot now.”

**Answer.** *False. It is not dominant since, if the other player were not to shoot next turn, you would do better to wait and get a better shot at your next turn.*

- (d) [5 points] “Lowering the tuition to go to elite schools like Harvard and Yale makes it harder for bright students to distinguish themselves from less bright students.”

**Answer.** *False. The use of schools like Harvard and Yale as signals depends on their being a cost difference between bright and less bright students. The tuition is a symmetric cost across students of different abilities.*

## USE BLUE BOOK 1

USE BLUE BOOK 2

Question 2. [15 total points]

Two players,  $A$  and  $B$  play the following game. First  $A$  must choose IN or OUT. If  $A$  chooses OUT the game ends, and the payoffs are  $A$  gets 2, and  $B$  gets 0. If  $A$  chooses IN then  $B$  observes this and must then choose in or out. If  $B$  chooses out the game ends, and the payoffs are  $B$  gets 2, and  $A$  gets 0. If  $A$  chooses IN and  $B$  chooses in then they play the following simultaneous move game:

		$B$	
		left	right
$A$	up	3, 1	0, -2
	down	-1, 2	1, 3

(a) [5 points] Draw the tree that represents this game?

**Answer.** See attached figure.

(b) [10 points] Find all the *pure-strategy* SPE of the game.

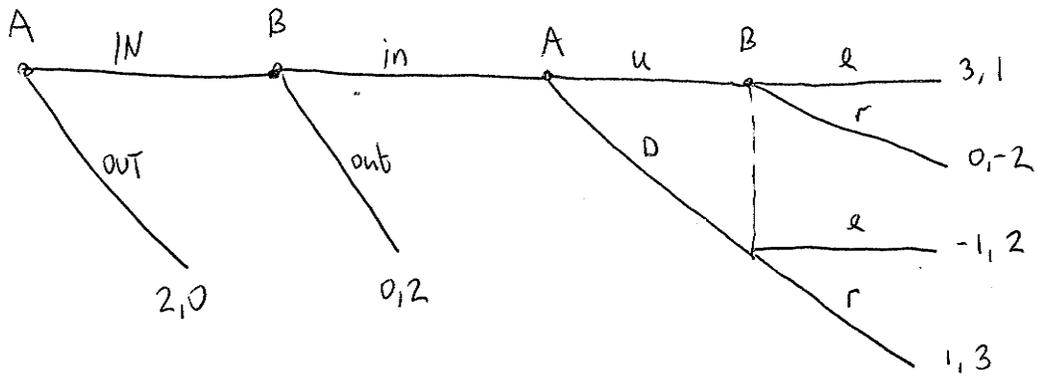
**Answer.** In the last subgame (the one represented by the matrix above), there are two pure strategy equilibria (*up, left*) and (*down, right*). Each corresponds to an SPE of the whole game. The SPE are:

$$[(OUT, up), (out, left)] \text{ and } [(OUT, down), (in, right)]$$

USE BLUE BOOK 2

Figure for Answer 2(a)

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**USE BLUE BOOK 3**

**Question 3. [40 total points] Poverty Traps.**

Alex is deciding whether or not to make a loan to Brian who is very poor and who has a bad credit history. Simultaneous to Alex making this decision, Brian must decide whether or not to buy gifts for his grandkids. If he buys gifts, he will be unable to repay the loan. If he does not buy gifts, he will repay the loan. If Alex refuses to give Brian a loan, then Brian will have to go to a loan shark.

The payoffs in this game are as follows: if Alex refuses to make a loan to Brian and Brian buys gifts then both Alex and Brian get 0. If Alex refuses to make a loan to Brian and Brian does not buy gifts then Alex gets 0 and Brian gets  $-1$ . If Alex makes a loan to Brian and Brian buys gifts then Alex gets  $-2$  and Brian gets 7. If Alex makes a loan to Brian and does not buy gifts, then Alex gets a payoff of 3 and Brian gets a payoff of 5.

**(a) [5 points]** Suppose this game is played just once. Find the equilibria of the game.

**Answer.** *The matrix is shown below with BR shown by underlining*

		<i>B</i>	
		repay	not
<i>A</i>	Loan	<u>3</u> , 5	-2, <u>7</u>
	Not	0, -1	0, <u>0</u>

*There is only one NE, (Not, not). Since not is dominant for B there is no other NE.*

Now suppose that the game is repeated. Suppose that (for all players) a dollar tomorrow is worth  $2/3$  of a dollar today. In addition, suppose that, after each period (and regardless of what happened in the period), Brian has a  $1/2$  chance of escaping poverty. Assume that, if Brian escapes poverty then he will not need a loan from either Alex or a loan shark: if effect, Brian will exit the game. Assume that, if Brian escapes poverty, he will never return. Thus, after each period, there is only  $1/2$  chance of the game continuing. Given this, the effective discount factor for the game between Alex and Brian is  $(1/2) \times (2/3) = (1/3)$ .

Consider the following strategy profile. In period one, Alex makes Brian a loan. Thereafter, Alex continues to make Brian loans (if he is still poor) as long as Brian and has always got a loan and repaid it in the past. But if Brian ever does not repay (or does not get a loan) then Alex never makes a loan to Brian again. In period one, Brian does not buy gifts (and hence repays the loan if he gets one). Thereafter (as long as he is still poor), Brian does not buy gifts (and hence repays the loan if he gets one) as long as he has always got a loan and repaid it in the past. But if Brian ever does not repay (or does not get a loan) then he will return to buying gifts and hence never repay a loan again.

**(b) [12 points]** Is this strategy profile an SPE of the repeated game?

**Answer.** *In this strategy profile, regardless of the history, Alex is always playing a stage-game BR to Brian's equilibrium action, and no change in Alex's choices ever makes Brian's equilibrium future actions 'improve' from Alex's point of view. Hence Alex has no incentive to deviate. Similarly, where the supposed equilibrium instructs Alex to refuse to make loans to Brian for ever, it instructs Brian not to repay. Since Brian is playing a stage-game BR to Alex's equilibrium action and since no change in Brian's choices induces any change in Alex's actions, Brian has no incentive to deviate from this.*

However, where the equilibrium specifies that Brian is supposed to repay, he has a temptation to buy gifts and hence not repay. The incentive equation is

$$(7 - 5) \stackrel{?}{\leq} \delta \left( \frac{5}{1 - \delta} - \frac{0}{1 - \delta} \right)$$

which reduces to  $\delta \geq 2/7$ . But the effective  $\delta = 1/3 > 2/7$ . Hence Brian has an incentive to repay.

**(c) [8 points]** Suppose that the government introduces regulation of loan sharks. As a consequence, Brian's payoff in each period in which he still needs a loan but does **not** get it from Alex is 1 if he does not buy gifts and 2 if he buys gifts. Explain whether or not this policy is likely to be good for Brian.

**Answer.** The policy undermines Brian's incentive to repay in the proposed equilibrium above. The incentive equation now reads

$$(7 - 5) \stackrel{?}{\leq} \delta \left( \frac{5}{1 - \delta} - \frac{2}{1 - \delta} \right)$$

which reduces to  $\delta \geq 2/5$ , but the effective  $\delta = 1/3 < 2/5$ . Hence Brian will not repay. Thus, the strategy profile above is no longer an SPE. Brian will have to go to a loan shark for an equilibrium payoff (in poverty) of  $\frac{2}{1 - \delta} = 3$ . Previously, he had an equilibrium payoff (in poverty) of  $\frac{5}{1 - \delta} = 7.5$ .

**(d) [8 points]** Suppose that the government abandons its loan-shark policy and replaces it with a job scheme that increases the probability after each period of Brian escaping poverty to  $2/3$  (i.e.,  $1/3$  chance of returning to the loan game). Explain the likely consequences of this policy for the business relationship between Alex and Brian.

**Answer.** The effective discount factor for the loan game is now  $(1/3) \times (2/3) = (2/9)$ . Looking back at the first incentive equation above, we see that, since  $2/9 < 2/7$ , Brian will not repay the loan and the relationship between Alex and Brian will break down.

**(e) [7 points]** [Harder] For the policy in part (d) what extra information would you need to know whether this policy is good or bad for Brian (ignoring the welfare of Alex or Brian's grandkids). Explain as carefully as you can. **[Do not spend all your time on this: you can come back later.]**

**Answer.** The key missing piece of information is that we do not know the payoff (the value) of being out of poverty. Brian's is now worse off in poverty because he has to go to a loan shark, but he is more likely to escape poverty and get a higher non-poor payoff.

Let  $V$  be the value of not being poor. Let the true discount factor be  $\delta = 2/3$ . Before, the expected NPV from not-being-poor was

$$\frac{1}{2}\delta V + \frac{1}{2}\frac{1}{2}\delta^2 V + \dots = \delta V \left( \frac{1/2}{1 - \delta/2} \right).$$

Now it is

$$\frac{2}{3}\delta V + \frac{1}{3}\frac{2}{3}\delta^2 V + \dots = \delta V \left( \frac{2/3}{1 - \delta/3} \right)$$

Plugging in  $\delta = 2/3$ , the difference between these two is:

$$\frac{2}{3}V \left( \frac{2/3}{7/9} - \frac{1/2}{2/3} \right) = \frac{V}{14}$$

The difference in expected NPV within poverty is

$$\frac{5}{1 - 1/3} - \frac{0}{1 - 2/9} = \frac{15}{2}$$

So, for the new policy to benefit Brian we need  $V > 15 \times 7$ . If we think of the per period welfare of the non-poor as  $w$  then this becomes  $w/(1 - \delta) > 15 \times 7$  or (using  $\delta = 2/3$ )  $3w > 15 \times 7$  or  $w > 35$ . Hence, we can see that we need the non-poor to be doing quite well for this policy to be good for Brian.

**USE BLUE BOOK 3**

USE BLUE BOOK 4

Question 4. [30 total points] “Exclusive”.

The Europa Club has a formal procedure (which we can think of as a game) to select its members. At each stage of the game, the ‘newest member’ to have been admitted into the club can either declare the membership-game over or nominate a new candidate to become a member. If a candidate is nominated, the existing members of the club vote whether to admit or reject. If the candidate is rejected, then the membership game is over. If the candidate is admitted, then the game continues with the now-admitted candidate becoming the ‘newest member’ choosing whether to nominate someone or end the game. The final membership of the club are the members when the game is over.

Whenever votes occur, the voting rules are as follows. The existing members vote sequentially, starting with the newest member (assume the nomination is his vote) and ending with the first member. The candidate does **not** get a vote. All votes are observed by everyone. If the candidate gets a half or more of the votes, she is admitted. That is, if there is a tie, then the candidate is admitted. There are no abstentions. Once you become a member, you are a member for ever: you cannot be voted off and you cannot leave.

Suppose initially that  $A$  is the only member of the club (and hence also its newest member). There are only three possible other members:  $B$ ,  $C$  and  $D$ . Thus, in the first stage of the game,  $A$  can either nominate one of these as a candidate (and then ‘vote’ them in), or end the game and remain alone.

The following table gives the preferences of each possible member over possible final memberships of the club.

$A$	$B$	$C$	$D$
$ac$	$abcd$	$acd$	$abd$
$ab$	$ab$	$ac$	$ad$
$ad$	$abd$	$abcd$	$acd$
$a$	$abc$	$abc$	$abcd$
$abcd$	$a$	$a$	$a$
$abc$	$ac$	$ab$	$ab$
$acd$	$ad$	$ad$	$ac$
$abd$	$acd$	$abd$	$abc$

Thus, for example,  $B$ ’s most preferred final membership would have everyone in the club. Her second preference would be just  $A$  and herself. Her third preference would be  $A, D$  and herself. And her fourth preference would be  $AC$  and herself. All other memberships rank lower in her preferences.

(a) [5 points] Suppose three candidates have been admitted, and a fourth has been nominated. How will player  $A$  vote? Explain why this means that any nominated member will be admitted.

**Answer.** *Player  $A$  prefers  $abcd$  to any club of three members so he will always vote yes if we get there. Given this, the voting is always trivial. In the first round, the nominator is the only voter. In the second, the nominator provides one vote and that is all that the candidate needs. And in the third round the nominator plus player  $A$  provide two votes which is a majority.*

(b) [25 points] Assuming that all members have taken game theory, explain carefully how you would expect the game to proceed. [Most of the points are for the explanation.]

**Answer.** *A tree would work but try this. Suppose A nominates C so C finds herself in a club of ac. The only outcome that player C prefers to ac is acd, so the only nomination she would consider is d or “stop”. If she chooses d, then D finds himself in a club of acd. The only memberships D prefers to this are no longer feasible (since he cannot kick anyone out). So he stops. But this is good for C so she will indeed nominate D but this is bad for A since he preferred a to acd. So A will not nominate C.*

*Next suppose A nominates B. Then B finds himself in a club of ab. He would prefer to get to abcd. If B nominates D then D finds himself in a club of abd which is his most preferred club. So D would stop. This is bad for B so B won't nominate D. But if B nominates C, then C finds herself in a club of abc. She would prefer abcd so she nominates D. This is good for B (who got to abcd) so B would nominate C. But this is bad for A who preferred a to abcd so A won't nominate B.*

*Next suppose that A nominates D. Then D finds himself in a club of ad. The only club D would prefer would be abd. But if D nominates B, B will nominate C to get to abcd. But D prefers ad to abcd, so D won't nominate B. Instead, D will stop. But this outcome, ad, is better for A than a so A nominates D (who stops)!*

USE BLUE BOOK 4

Question 5. [45 total points] “Paid in the USA” [Notice that you do not need to know any auction theory to answer this question.]

Two interest groups, A and B, are lobbying congress about an upcoming bill. Everyone knows that it is worth \$3M to A to get the bill passed into law, and it is worth \$2M to B to get the bill to fail. Congress decides to sell off its vote using a sealed-bid second-price auction. That is, A and B simultaneously write down a ‘bid’. The bids,  $b_A$  and  $b_B$  are then ‘opened’. If  $b_A \geq b_B$  then congress passes the bill and A must pay  $b_B$  to the “congressmen’s fund”. If  $b_B > b_A$  then congress rejects the bill and B must pay  $b_A$  to the “congressmen’s fund”. Notice: if the bids are tied then A ‘wins’; **the winner pays the loser’s bid**; and the loser pays nothing.

(a) [10 points] Recall from class that bidding your value is a weakly dominant strategy in a second-price auction. Argue carefully but concisely that, for A, bidding \$3M weakly dominates bidding \$2.8M.

**Answer.** *The only occasions that bidding 3 and bidding 2.8 lead to different outcomes is if  $2.8 < b_B \leq 3$ . And the only case in which they lead to different payoffs is if  $2.8 < b_B < 3$ . In this case,  $b_A = 3$  leads to winning the auction and a payoff of  $3 - b_B > 0$ . Bidding 2.8 leads to a payoff of 0. Hence bidding 3 weakly dominates bidding 2.8.*

(b) [5 points] Assuming that no-one chooses a weakly dominated strategy, what are the equilibrium bids, payments and payoffs in the auction?

**Answer.** *The bids are 3 and 2, so A wins and pays 2 to congress. The bidders’ payoffs are (1, 0).*

Now suppose that, if and only if congress passes the bill, it goes to the president. If the president signs the bill, it passes into law. If he vetoes it, it fails. The president decides that, if the bill gets to him, he will also hold a sealed-bid, second-price auction under exactly the same rules except that payments are made to the “president’s fund”. That is, there are potentially two auctions, held sequentially. The first auction decides whether or not congress passes the bill with payments made to congress accordingly. Then, afterwards (if congress passes the bill), a second, **new** auction decides whether or not the president signs the bill with payments to the president accordingly.

(c) [5 points] Suppose that A wins the first auction, and the bill passes congress (with A paying, say,  $b_B = \$0.9M$  to the congressmen’s fund). Assuming that no-one chooses a weakly dominated strategy in the subsequent presidential auction, what are the equilibrium bids, payments and continuation payoffs in that auction?

**Answer.** *Since the first payment of  $b_B = \$0.9M$  to the congressmen’s fund is sunk, the auction is the same as in part (b). The bids are 3 and 2, so A wins and pays 2 to the president. The bidders’ continuation payoffs (net of the initial 0.9) are (1, 0).*

(d) [7 points] Assuming that no-one chooses a weakly dominated strategy in any subgame, explain the SPE outcome of the whole game including the first-stage bids and payments.

**Answer.** *The continuation payoffs from stage two effect the ‘values’ for stage one. For A, the continuation equilibrium value of winning the first stage is 1 while the value of losing is 0. So A bids his ‘value’ of 1. For B the equilibrium continuation value of losing the first stage is 0 while the value of winning is 2. So B bids her value of 2. B wins and pays 1 to congress.*

Now suppose that congress (realizing that A is deterred from bidding much in the first auction) makes the following offer only to A. If A ‘wins’ the congressional auction (so that

congress passes the bill and A pays B's bid  $b_B$  to congress) but the bill is then vetoed by the president (that is, A loses the second auction), then congress will refund to A its payment  $b_B$  minus a small processing fee.

(e) [10 points] Suppose that A wins the first auction, and the bill passes congress with A paying  $b_B = \$0.9M$  to the congressmen's fund. Assuming that no-one chooses a weakly dominated strategy in the subsequent presidential auction, what are the equilibrium bids and payments in that auction? How would your answer change if A had paid  $b_B = \$1.1M$  in the first stage?

**Answer.** Let the processing fee be  $\varepsilon (< 0.1M)$ . Since the first payment of  $b_B = \$0.9M$  to the congressmen's fund is not sunk, the value to player A of winning the second auction is approximately  $3 - (b_B - \varepsilon)$ . Thus, A bids  $3 - b_B + \varepsilon$ , and B bids her value 2. The outcome depends then on  $b_B$ . For  $b_B = \$0.9M$ , A wins and pays 2 to the president (and  $\$0.9$  to congress). For  $b_B = \$1.1$ , B wins and pays  $1.9 + \varepsilon$  to the president (and A pays only the processing fee  $\varepsilon$  to congress).

(f) [8 points] [Harder] Assuming that no-one chooses a weakly dominated strategy in any subgame, explain the SPE outcome of the whole game including the first-stage bids and payments.

**Answer.** The (undominated) SPE outcome is for A to bid 1, B to bid more than 1, and for B to win the first auction and pay 1 to congress. Notice this is the same outcome as in part (d).

First, notice that A will only win the second stage if he wins the first stage and pays  $b_B$  less than or equal  $1 + \varepsilon$  in that first stage. That is, in stage two, player B bids 2 and player A bids  $3 - b_B + \varepsilon$ . This is the (undominated) NE of the second stage game.

I claim that A should bid 1 in the first stage. Suppose that A does bid 1 in the first stage. Then, if he wins the first stage,  $b_B \leq 1$ . Thus A will bid  $3 - b_B + \varepsilon > 2$  in the second stage, win that stage too, and end up with a total payoff of  $3 - 2 - b_B \geq 0$  (in fact, this is greater than 0 unless  $b_B = 1$  exactly). If  $b_B > 1$  then A loses the first auction and pays nothing.

Now suppose that, instead, player A bids  $b_A^L < 1$  in the first stage. The only way in which this can lead to a different final payoff for A than the payoff from bidding 1 is if  $b_A^L < b_B < 1$ . In these cases, bidding  $b_A^L$  yields zero but bidding 1 yields a positive payoff.

Next consider player A's bidding  $b_A^H > 1$  in the first stage. If  $b_B < 1$  then this bid leads to exactly the same outcome as bidding 1: both bids lead to A winning both auctions and ending up with the same final payoff of  $3 - 2 - b_B > 0$ . If  $b_B = 1$  then bidding either  $b_A^H$  or 1 leads to A winning both auctions and paying a total of 2: both bids yield a final payoff of 0. If  $b_B > b_A^H$ , then bidding 1 or bidding  $b_A^H$  both lose the first auction and yield a payoff of zero. So the only cases that make a difference are if  $1 < b_B \leq b_A^H$ . In this case, bidding 1 leads A to lose the first auction and get a payoff of 0. But bidding  $b_A^H$  wins the first auction. There are then two cases: if  $1 < b_A^H \leq 1 + \varepsilon$  (and  $b_B \leq b_A^H$ ) then player A will win the second auction also and get a final payoff of  $3 - 2 - b_B < 0$ : worse than the payoff from bidding 1. The other case is if  $b_B > 1 + \varepsilon$  (and  $b_B \leq b_A^H$ ). In this case, A will lose the second auction and get a final payoff of  $-\varepsilon$ : worse than the payoff from bidding 1. Thus, bidding 1 'weakly dominates' all other bids.

Now that we know that A bids 1, we are almost done. Player B's best response to player A bidding 1 in the first stage is to bid  $b_B > 1$  to win the first stage. (Since  $b_A = 1$ , if B lost the first-stage auction by bidding  $b_B \leq 1$ , B would also lose the second-stage auction.) Hence the SPE outcome is for A to bid 1, B to bid more, and B to win the first auction and pay 1 to congress.

*You might wonder if  $B$  has a dominant strategy in the first auction. She does not. If  $A$  bids a crazy high bid, then  $B$  wants to bid the same, let  $A$  win the first auction and then have  $B$  win the second auction for next to nothing. But these crazy high bids for  $B$  lead to a bad outcome if  $A$  bids just a little less, making  $B$  win the first auction for a crazy amount of money.*

USE BLUE BOOK 5