

# Problem Set 3

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## The Linear City: Price Competition with Differentiated Products.

In class, we considered two models of duopoly competition: Cournot (quantity) competition and Bertrand (price) competition. It seems more realistic to think of firms' competing in prices than in quantities, but the Cournot outcome seems more 'realistic' than the Bertrand outcome. This problem considers a third model of duopoly competition. Like Bertrand, the two firms will compete in prices rather than quantities. Unlike the Bertrand model, however, the products of the two firms are not identical. In economist jargon, the products are differentiated. Instead of my solving the model on the board, you will solve it for yourselves. But don't panic: The problem set will walk you through this step by step. Answer all the numbered questions below.

### The Game.

- We can think a 'city' as a line of length one.
- There are two firms, 1 and 2, at either end of this line.
  - The firms simultaneously set prices  $p_1$  and  $p_2$  respectively.
  - Both firms have constant marginal costs,  $c$ .
  - Each firm's aim is to maximize its profit.
- Potential customers are evenly distributed along the line, one at each point.
  - Let the total population be one (or, if you prefer, think in terms of market shares).
- Each potential customer buys exactly one unit, buying it either from firm 1 or from firm 2. So total demand is always exactly one.

- Consider a customer at a position  $y$  on the line. She is distance  $y$  from firm 1 and distance  $(1 - y)$  from firm 2.

– The customer at position  $y$  on the line is assumed to buy from firm 1 if

$$p_1 + ty^2 < p_2 + t(1 - y)^2; \quad (\text{a})$$

to buy from firm 2 if

$$p_1 + ty^2 > p_2 + t(1 - y)^2; \quad (\text{b})$$

and to toss a fair coin if this is an exact equality.

**Interpretation.** Customers care about both price and about the ‘distance’ they are from the firm. If we think of the line as representing geographical distance, then we can think of the  $t \times (\text{distance})^2$  term as the ‘transport cost’ of getting to the firm. Alternatively, if we think of the line as representing some aspect of product quality — say, fat content in ice-cream — then this term is a measure of the inconvenience of having to move away from the customer’s most desired point. As the transport-cost parameter  $t$  gets larger, we can think of products becoming more differentiated from the point of view of the customers. If  $t = 0$  then the products are perfect substitutes.

### What happens?

- (1) Will either firm  $i$  ever set its price  $p_i < c$ ? Why?
- (2) Suppose that firm 2 sets price  $p_2$ . At what price can firm 1 capture the entire market (i.e., given  $p_2$ , at what  $p_1$  will all the customers will buy from firm 1)?

Let’s consider if Firm 1 can do better by setting a price higher than the solution to question (2). The downside of firm 1’s setting a higher price is that it will lose some of the market. The upside is that it will charge more to any customer it keeps. The next question gets you to work out just how many customers buy from firm 1 when the prices are ‘close’.

- (3) Suppose that prices  $p_1$  and  $p_2$  are close enough that the market is divided (not necessarily equally) between the two firms. Use expressions (a) and (b) above to find the location of the customer who is exactly indifferent between buying from firm 1 and buying from firm 2. Use your answer to argue that, when the market is split, firm 1’s demand is given by:

$$D_1(p_1, p_2) = \frac{p_2 + t - p_1}{2t} \quad (\text{c})$$

We now have all the information we need to calculate firm 1’s best response to each  $p_2$ . When the market is split, firm 1’s profits are given by

$$u_1(p_1, p_2) = p_1 D_1(p_1, p_2) - c D_1(p_1, p_2) \quad (\text{d})$$

where the first term is revenues and the second term is costs.

- (4) Use expressions (c) and (d), together with some simple calculus, to show that, for intermediate levels of  $p_2$ ,

$$BR_1(p_2) = \frac{p_2 + t + c}{2}$$

- (5) Draw a picture of the best responses of firms 1 and 2. Be careful to indicate in your picture what happens to  $BR_1(p_2)$  when  $p_2 < c - t$ , and when  $p_2 > 3t + c$ . [Hint: recall your answers to questions (1) and (2)]. Draw the best response  $BR_2(p_1)$  on the same picture.
- (6) Use algebra to find the Nash equilibrium.
- (7) What is the equilibrium price when  $t = 0$ . Interpret your answer. People sometimes say ‘competition gets less fierce as products become less similar and more differentiated’. How does this show up in our model?

### Some Lessons.

- (A) Firms like product differentiation. It allows them to charge higher prices and make higher profits. This argument is a little simple, however, since entry by new firms may bid these profits away.
- (B) A little realism can help our model. Here removing the extreme assumption in the Bertrand model that goods were perfect substitutes gave us an outcome model that seems more plausible.
- (C) The methods we have learned in class are quite powerful. This was a complicated enough model for it not to be immediately obvious what would happen. But, by simply going through the steps you learned in class (finding the best responses; finding where they ‘cross’ etc.), you were able to solve the model as a problem set!