Problem Set III
Fall 2006 Physics 200a
Figures at the end
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Many problems ask for answers in terms of symbols and not numbers. You may need to invoke things like g in the answer even if I did not explicitly define them.

1. Here is a problem from string theory for those of you seeking challenges. An object of mass m is hanging from two taut strings as shown in the figure (1). Find an expression for the tensions T_1 and T_2 on the two strings. Give the numerical values when $m = 10kg$, $a = \pi/6$ and $b = \pi/3$.

2. A block of mass m sits atop a mass M which rests on a frictionless table. The mass M is connected to a spring of force constant k attached to the wall. (i) How far can mass M be pulled so that upon release, the upper mass m does not slip off? The coefficient of friction between the two masses is μ. (ii) Repeat if μ' is the coefficient of friction between M and the table. See Figure (2).

3. A car moving at $70km/h$ collides rams into an immobile steel wall. Its front of the is compressed by $0.94m$. What average force must a seat belt exert in order to restrain a 75-kg passenger?

4. A 7.2-kg mass is hanging from the ceiling of an elevator by a spring of spring constant $150N/m$ whose unstretched length is 80 cm. What is the overall length of the spring when the elevator: (a) starts moving upward with acceleration $0.95m/s^2$; (b) moves upward at a steady $14m/s$; (c) comes to a stop while moving upward at 14 m/s , taking 9.0 s to do so? (d) If the elevator measures 3.2 m from floor to ceiling, what is the maximum acceleration it could undergo without the 7.2-kg mass hitting the floor?

5. A mass is hanging by a rope from the ceiling of a bus at rest. What angle does the rope make with the vertical when the bus accelerates by an amount a?

6. Two springs have the same unstretched length but different spring constants k_1 and k_2 . Find the effective force constant k_e if they are connected in series and in parallel. (To find k_e, imagine stretching the combination by an amount x and keeping track of the force needed.)

7. You stand on a merry-go-round spinning at f revolutions per second. You are R meters from the center. What is the minimum coefficient of static friction μ_s between your shoes and the floor that will keep you from slipping off?

8. A mass m undergoes circular motion of radius R on a horizontal frictionless table, connected by a massless string through a hole in the table to a second mass M

(Figure (3). If M is stationary, find (a) the tension in the string and (b) the period of circular motion.

9. A 1000-kg car is on a country road which has its ups and downs or maxima and minima in height. Right now it is at the top of a bump (a maximum) that has a radius of curvature of 20 m. What is the maximum speed at which the car can maintain road contact at the bump? If next it is at the bottom of a dip (a minimum) of the same radius, moving at the same speed, what force does the ground exert on the car?

10. The handle of a 22-kg lawnmower makes a 35° angle with the horizontal. If the coefficient of friction between lawnmower and ground is 0.68, what magnitude of force is required to push the mower at constant velocity? Assume the force is applied in the direction of the handle. Compare with the mower's weight.

11. A roller coaster of mass M is at the top of the Loop-the-loop of radius R at twice the minimum speed possible. What force does the track exert on it? What force does it exert when it is at the bottom of the circle? (Use conservation of energy if needed.)

12. Consider Figure (4) with μ_s the coefficient of static friction between M and the plane. Let masses initially be at rest. Imagine mass m is varied continuously to cause motion. Find the range of values of m for which (i) M will begin to slide down hill as m is reduced (ii) when M will begin to go uphill as m is increased. What happens in between these values? Argue that motion up or down is possible for m even in this range if we give the masses a little push. What is the range of m when such motion is possible? Why is it that if we increase m enough it will necessarily drag M up hill, but an M, no matter how large may not be able to cause m to go up?

13. I pull a mass m resting at $x = 0$ on a frictionless table connected to a spring with some k by an amount A and let it go. (i) What will be its speed at $x = 0$? (ii) How far to the left will it go and why? (ii) Repeat (i) when the coefficient of friction with table is μ_k (iV) Repeat (II) with friction.

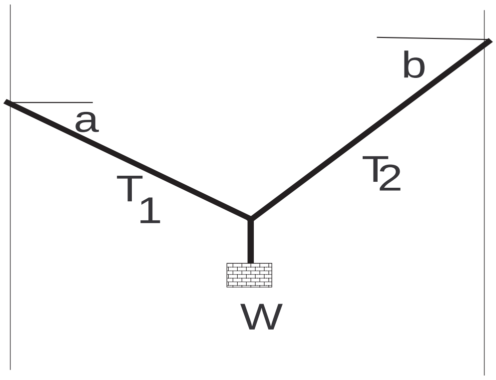

FIG. 1. Two strings making angles a and b meet at a point and support a mass 10 kg.

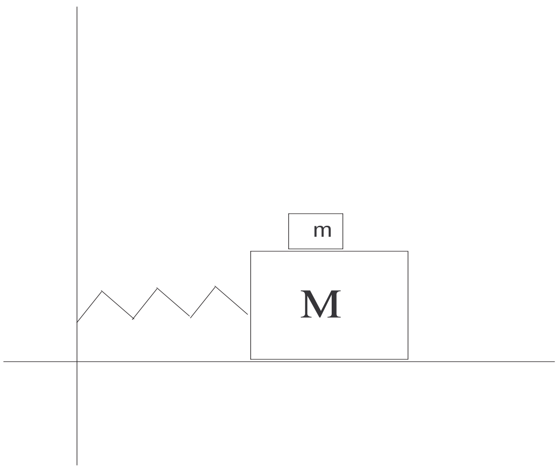

FIG. 2. Spring has force constant k. The coefficient of friction between masses is μ in both parts and is μ' between lower mass and table in part (ii)

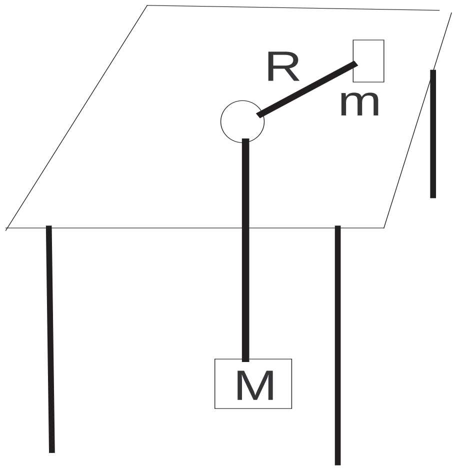

FIG. 3. The mass m is going around in a circle of radius R on a table. It is connected by a massless rope to M, hanging under the table.

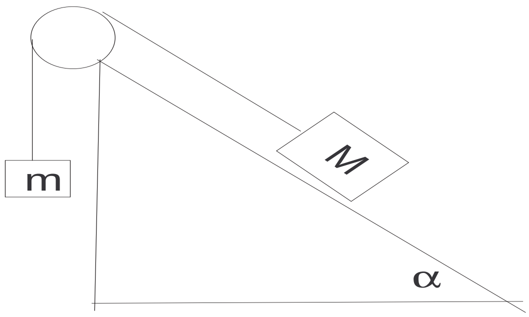

FIG. 4. The coefficient of static (kinetic) friction between M and plane is μ_s (μ_k).