Physics 200
Problem Set 1
Solution

Note: It's not very fun to punch numbers into a calculator. Plugging in numbers at the very end will often save you time and mistakes. This won't matter so much in this problem set, but try to get in the habit now.

1. From the top of a building of height $h = 100$ m I throw a stone up with velocity 10 m/s. What is the maximum height it reaches, and when does this occur? How many seconds does it spend on its way down between $h = 50$ m and $h = 0$ m? What is its velocity when $h = 50$ m? If, while the stone is airborne, an earthquake opens up a hole 50 m deep in the ground, when and with what speed will the stone hit the bottom?

   Answer: When the stone reaches its maximum height its velocity is momentarily zero. So

$$0 = v_0 - gt \implies t = \frac{v_0}{g} = \frac{10 \text{ m/s}}{9.81 \text{ m/s}^2} = 1.0 \text{ s},$$

   and

$$0 = v_0^2 - 2g\Delta h \implies \Delta h = \frac{v_0^2}{2g} = \frac{(10 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 5.1 \text{ m}.$$

   This means the actual height reached by the stone is

$$h_{\text{building}} + \Delta h = 105.1 \text{ m}.$$

   When the stone is at some height Δh measured from the top of the building its velocity is

$$v^2 = v_0^2 - 2g\Delta h,$$

   and between two heights Δh_1 and Δh_2

$$v_2 = v_1 - gt \implies t = \frac{v_1 - v_2}{g}.$$

   Plugging in the numbers gives (negative because the stone is moving downward)

$$v_1 = -\sqrt{v_0^2 - 2g\Delta h_1} = -\sqrt{(10 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(50 \text{ m})} = -32.9 \text{ m/s},$$
$$v_2 = -\sqrt{v_0^2 - 2g\Delta h_2} = -\sqrt{(10 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(100 \text{ m})} = -45.4 \text{ m/s}$$

   and

$$t = \frac{-32.9 \text{ m/s} - (-45.4 \text{ m/s})}{9.81 \text{ m/s}^2} = 1.3 \text{ s}.$$

   The velocity when $h = 50$ m is -33 m/s.

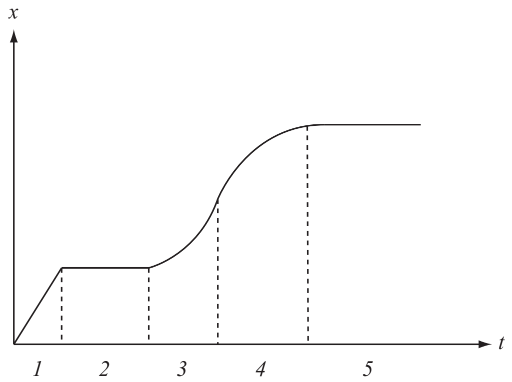
   Using the same equations again, when the stone hits the bottom of the hole

$$v = -\sqrt{v_0^2 - 2g\Delta h_{\text{hole}}} = -\sqrt{(10 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(150 \text{ m})} = -55.2 \text{ m/s}.$$

   The time it takes to get here is

$$t = \frac{10 \text{ m/s} - (-55.2 \text{ m/s})}{9.81 \text{ m/s}^2} = 6.6 \text{ s}.$$

2. Below is the plot of position vs. time for a car. Explain what the car is doing in each numbered interval.



Answer: The car is moving at constant velocity in 1 until it stops abruptly and stays still in 2. It then accelerates (more or less constantly) in 3, then decelerates in 4 until it comes to a stop and stands still in 5.

3. Romeo is at $x = 0$ m at $t = 0$ s when he sees Juliet at $x = 6$ m.

(a) Romeo begins to run towards her at $v = 5$ m/s. Juliet, in turn, begins to accelerate towards him at $a = -2$ m/s^2. When and where will they cross? Sketch their motions by measuring time on the horizontal axis and position on the vertical axis.

(b) Suppose, instead, that Juliet moved away from Romeo with *positive* acceleration a. Find a_{max}, the maximum acceleration for which Romeo can catch up with her. For this case find the time t of their meeting. Show that for smaller values of a these star-crossed lovers cross twice. Draw a sketch for this case. Explain in words why they cross twice.

Answer:

(a) Let R stand for Romeo and J for Juliet. Then

$$x_R = vt, \qquad x_J = x_0 + \frac{1}{2}at^2.$$

When they meet,

$$vt = x_0 + \frac{1}{2}at^2 \implies \frac{1}{2}at^2 - vt + x_0 = 0.$$

The solution is

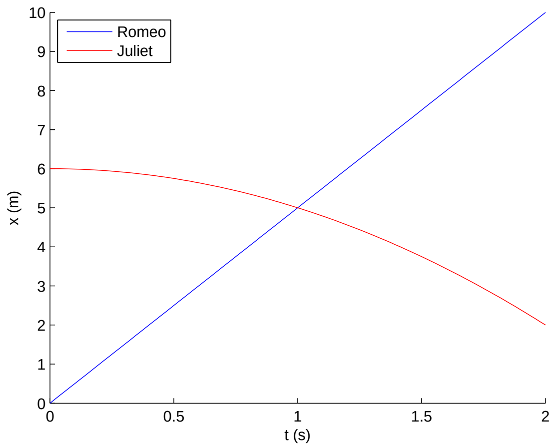
$$t = \frac{v \pm \sqrt{v^2 - 2ax_0}}{a} = \frac{v}{a}\left(1 \pm \sqrt{1 - \frac{2ax_0}{v^2}}\right).$$

Keeping in mind that a is negative, taking the *minus* sign for t gives the positive solution we want:

$$t = \frac{5 \text{ m/s}}{-2 \text{ m/s}^2}\left(1 - \sqrt{1 - \frac{2(-2 \text{ m/s}^2)(6 \text{ m})}{(5 \text{ m/s})^2}}\right) = 1 \text{ s}.$$

At this time

$$x_R = x_J = vt = (5 \text{ m/s})(1 \text{ s}) = 5 \text{ m}.$$



(b) The same equations apply, but we see that if

$$1 - \frac{2ax_0}{v^2} < 0$$

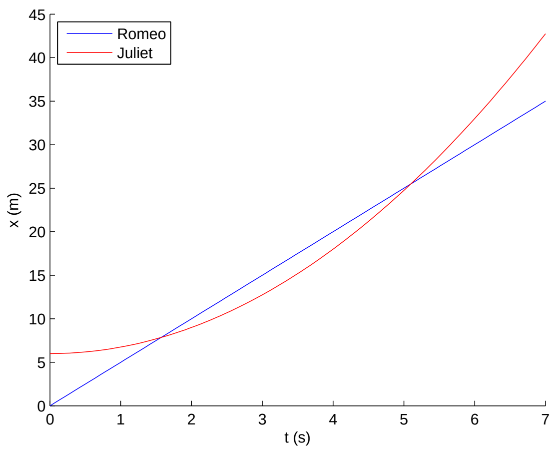
there are no solutions for t. So setting this equal to zero we find

$$a_{\max} = \frac{v^2}{2x_0} = \frac{(5 \text{ m/s})^2}{2(6 \text{ m})} = 2.1 \text{ m/s}^2.$$

If $a = a_{\max}$ then the time of their (one-time) meeting is simply

$$t = \frac{v}{a} = \frac{5 \text{ m/s}}{2.1 \text{ m/s}^2} = 2.4 \text{ s}.$$

We see from the above that if a is positive and such that the quantity inside the square root is positive, then it is also less than 1 and hence there are 2 positive solutions for t. This means Romeo and Juliet meet twice. The plot below is for $a = 1.5$ m/s^2.



What happens is that, since Juliet starts from rest, if her acceleration is not too great Romeo can immediately catch up with her and move *ahead* of her. Then Juliet starts to acquire more and more speed and eventually catches up with Romeo. After the second meeting Romeo keeps trudging along at the same velocity, but Juliet keeps speeding up so that Romeo will never catch up with her again. Aye them.

4. A particle moves according to the equation $x = 3+4t+6t^2+4t^3$. Find its velocity and acceleration at all times. When does its velocity equal 10 m/s? What is its acceleration at that instant?

   Answer: Here we assume that t is given in seconds and x in meters, so that v is m/s and a is m/s^2.

$$v = \frac{dx}{dt} = 4 + 12t + 12t^2,$$
$$a = \frac{dv}{dt} = 12 + 24t.$$

   For a given v we have

$$12t^2 + 12t + 4 - v = 0 \implies t^2 + t + \frac{4-v}{12} = 0,$$

   so the quadratic formula gives

$$t = \frac{-1 \pm \sqrt{1-(4-v)/3}}{2},$$

   and for $v = 10$ we have

$$t = \frac{-1 + \sqrt{1-(4-10)/3}}{2} = 0.37 \text{ s},$$

   where we take the positive sign as usual. The acceleration at this time is

$$a = 21 \text{ m/s}^2.$$

5. (Difficult) Ball A is dropped from rest from a building of height H exactly as ball B is thrown up vertically from the ground. When they collide A has twice the *speed* of B. If the collision occurs at height h, what is h/H? *Hint:* Write equations for heights y_A, y_B and velocities v_A, v_B. What can you say about them at the time of the collision?

   Answer: In this problem we will use all three of our main equations for motion in one dimension. If we let the ground be $y = 0$, then

$$y_A = H - \frac{1}{2}gt^2, \qquad y_B = v_0 t - \frac{1}{2}gt^2.$$

   At the moment that they collide $t = t_c$ and $y_A = y_B = h$, which means

$$H = v_0 t_c.$$

   Next

$$v_A^2 = -2g(y_A - H), \qquad v_B^2 = v_0^2 - 2gy_B,$$

   and when the balls collide $y_A = y_B = h$ and in addition $v_A^2 = 4v_B^2$, which gives

$$-2g(h-H) = 4(v_0^2 - 2gh) \implies v_0^2 = \frac{gH + 3gh}{2}$$

Finally

$$v_A = -gt, \qquad v_B = v_0 - gt,$$

but again $t = t_c$ and $|v_A| = 2|v_B|$ implies

$$gt_c = 2(v_0 - gt_c) \implies t_c = \frac{2v_0}{3g},$$

where we have made the reasonable assumption that the collision occurs while ball B is moving upward. Putting all three results together, we get

$$H = v_0 t_c = \frac{2v_0^2}{3g} = \frac{1}{3}H + h,$$

or

$$\frac{h}{H} = \frac{2}{3}.$$