Physics 200a Relativity crib sheet Shankar (2006)

Here are some very basic things you should know to do this problem set.

$$X = (ct, x) = (x_0, x_1)$$

Under a LT

$$x_1^{'} = \frac{x_1 - \beta x_0}{\sqrt{1-\beta^2}} \tag{1}$$

$$x_0^{'} = \frac{x_0 - \beta x_1}{\sqrt{1-\beta^2}}, \tag{2}$$

$$x_2^{'} = x_2 \tag{3}$$

$$x_3^{'} = x_3 \tag{4}$$

where $\beta = \frac{u}{c}$. Let us forget x_2, x_3.

Note that we use $x_0 = ct$ and not just t since x_0 has the same units as x and the LT looks nice and symmetric. You can check that

$$X \cdot X \equiv X^2 = x_0^2 - x_1^2 = x_0^{'2} - x_1^{'2} = s^2$$

is the same for all observers, an invariant.

A particle of mass m and velocity v has energy E and momentum p given by

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \qquad p = \frac{mv}{\sqrt{1 - v^2/c^2}} \tag{5}$$

The energy-momentum vector is

$$P = (P_0, P_1) = (\frac{E}{c}, p)$$

It is a pity we need to bring in $\frac{E}{c}$ and not E but this is to make sure both components have the same units (recall $x_0 = ct$) and LT has the same form as for components of X:

$$P_1^{'} = \frac{P_1 - \beta P_0}{\sqrt{1-\beta^2}} \tag{6}$$

$$P_0^{'} = \frac{P_0 - \beta P_1}{\sqrt{1-\beta^2}}, \tag{7}$$

It follows

$$P^2 = P_0^2 - P_1^2 = P_0'^2 - P_1'^2$$

is an invariant. What is this invariant value? You can show by explicit calculation using Eq. (5) that

$$P^2 = m^2c^2$$

Or you can be clever and say that since it is invariant, I will calculate it in the frame moving with the particle. There only $P_0 = mc$ is nonzero and the result follows. We can also rewrite

$$P^2 = (E/c)^2 - p^2 = m^2c^2$$

as

$$E^2 = (cp)^2 + m^2c^4.$$

Photons also have energy, which we denote by ω instead of E, and momentum, which we denote by k instead of p. We assemble these onto a four-vector

$$K = (K_0, K_1) = (\omega/c, k)$$

whose components obey

$$\omega = kc$$

This is the same as

$$K \cdot K \equiv K^2 = 0.$$

In other words $P^2 = m^2c^2$ becomes $K^2 = 0$ when applied to massless particles like photons. However massless does not mean momentum-less or energy-less.

In any collision, you set the sum of initial four moments equal to the sum of the final four-momenta. This is really four equations (or two if we set motion along y and z to zero) and sometimes rather than juggle these equations you should think in terms of four vectors and their dot products, as illustrated in class and the notes.

Remember P^2 same in all frames whether it refers to the momentum of one particle or the sum over many. When P is the momentum of a single particle $P^2 = c^2m^2$ regardless how it is moving. When P is the sum of many momenta, such as total of all incoming momenta, P^2 can by anything, but the same anything for all observers. (Recall the anti-proton creation experiment where the square of the total momentum was evaluated in the CM frame).