Problem Set IV- Assign Sep 27, 2006
Fall 2006 Physics 200a
Figures at the end
R. Shankar

Many problems ask for answers in terms of symbols and not numbers. You may need to invoke things like g in the answer even if I did not explicitly define them.

1. (i) Using the dot product show that if $\mathbf{A} + \mathbf{B}$ is perpendicular to $\mathbf{A} - \mathbf{B}$, then $A = B$.(ii) Use the dot product to relate vector components $(A_x,\ A_y)$ in the standard frame to $A'_x,\ A'_y)$ measured in a frame rotated counterclockwise by an angle θ. Use the fact that components of a vector are just dot products with the corresponding basis vectors: $A_x = \mathbf{A} \cdot \mathbf{i}$ etc.

2. If $F(x) = Ax^{3/2}$ find the work done by it in moving a body from $x = 0$ to $x = A$.

3. You have to choose between a 225-W fridge that costs \$ 1150 and a 425-W model that costs \$ 850. The first runs 11% of the time and the second runs 20% of the time. If electricity costs 9.5 cents/kWh when will you break even? (Use hours as the unit of time at first, go to days to report final answer.)

4. See the masses in Figure (1) which start out at rest. (i) Find the velocity of the 14 kg mass just before it hits the ground. (ii) Find the maximum height reached by the 8 kg (and don't worry about hitting the pulley). (iii) Find the fraction of mechanical energy left when the system finally comes to rest.

5. A loop-the-loop car of mass m comes down from a height h. (i) What is the minimum h (in terms of the loop radius R) that will ensure a safe trip? (ii) What will be the speed of the car at the lowest part of the circle? (iii) What will be the force exerted by the track on the car at this point? (iv) If the track after the loop is horizontal and has a coefficient of friction μ_k how far will the car travel before stopping? (v) If instead of using gravity, a spring of force constant k gives it the initial speed by how much should it be compressed? Get the actual numbers for part (v) if $m = 840kg, R = 6.2m, k = 31kN/m$.

6. A block of mass m comes down an inclined plane at angle at $\pi/6$ from a height h after being given a slight tap to get it going. If the coefficient of friction is μ_k what is the velocity when it reaches ground? Find this using Newton's Law as well as energy principles.

7. A mass m falls from a height h on to a spring of force constant k. Show that the maximum spring compression is $(mg/k)(1 + \sqrt{1 + 2kh/mg})$.

8. Find the work done by a force $\mathbf{F} = \mathbf{i}x^2y^3 + \mathbf{j}x^3y^2$ on going from from $(0,0)$ to $(1,1)$ along three paths: (i) first along x and then along y (ii) at 45^0 (iii) along the curve $y = x^2$. (iv) Show that this is a conservative force and find the potential energy $U(x, y)$. Find the work done earlier in terms of difference in U. (v) Repeat parts (i) and (ii) for $\mathbf{F} = \mathbf{i}xy^3 + \mathbf{j}xy$

9. Show that the acceleration due to gravity at a height h above the earth can be approximated (for small h) by $g_h = (1 - 2h/R_E)g_0$ where g_0 is just g and R_E is the radius of the earth. What is g_h on top of Mount Everest? Hint: Use the very useful result $(1 + x)^n \simeq 1 + nx$ for $x << 1$.

10. Find the period of the circular orbit of our sun around the center of our Galaxy (take as point mass $4 \cdot 10^{41}$ kg) at a radius $3 \cdot 10^4$ light years.

11. A meteorite $80,000km$ from the earth is moving towards the earth at 2000m/s. Ignoring air friction what will be its velocity on impact?

12. Calculate the radius of the geosynchronous orbit about the earth.

13. A rocket is launched at escape velocity from the surface of the earth (radius R_E). What is its velocity when it is at a distance r from the center of the earth in terms of G and M_E?

14. A satellite is in an elliptical orbit around the earth with altitudes ranging from 230 to 890 km. At the high point it is moving at 7.23 km/s. What is its speed at the low point?
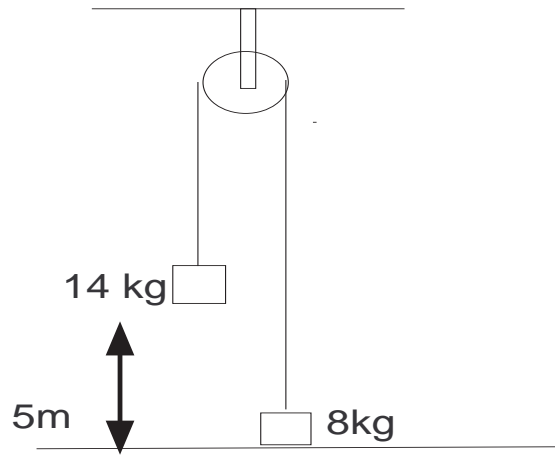
FIG. 1. The pulley is massless. Initially and finally all masses are at rest.