Problem Set V- Assign October 2, 2006 Due October 9.
Fall 2006 Physics 200a
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Many problems ask for answers in terms of symbols and not numbers. You may need to invoke things like g in the answer even if I did not explicitly define them.

1. Consider masses m_1, m_2, m_3 at x_1, x_2, x_3. Find X, the CM coordinate by finding X_{12}, the CM of mass of 1 and 2, and combining it with m_3. Show this is gives the same result as
$$X = \frac{\sum_{i=1}^{3} m_i x_i}{\sum_{i=1}^{3} m_i}$$

2. Consider a square of mass 4 kg, side 2m, negligible thickness, with its sides oriented along the usual axes with its center at $(0, 0)$. (i) Determine its CM using symmetry arguments. (ii) Imagine that the 1m $\times$ 1m part of it in the fourth quadrant is chopped off. Where is the new CM? Do this using the extension of result in previous problem. Repeat using the following trick: view the chopped off shape as the full square plus a 1m $\times$ 1m square of *negative* mass -1kg in the fourth quadrant. (iii) A disk of radius R centered at the origin has a circular hole of radius $R/2$ centered at $(x = -R/2, y = 0)$. Where is its CM?

3. Ideal Zorro (mass M, no height) swings down on a vine of length L from a height H and grabs a kid of mass m (zero height, standing on the ground) and together they barely reach safety at a height h. Relate H to the other parameters. Give H in meters if $L = 40m, M = 100kg, h = 6m, m = 30kg$.

4. Two identical stars of mass M orbit around their CM. Show that
$$T^2 = \frac{2\pi^2 R^3}{GM}$$
where R is the distance between planets. (Draw a figure and keep track of the gravitational force on either star as well as its centripetal acceleration.) (ii) Repeat for two unequal masses m and M and show
$$T^2 = \frac{4\pi^2 R^3}{G(M+m)}$$
where R is the separation between stars.

5. Consider a massless boat of length L on frictionless water. At the left end is a person P_1 of mass m_1 holding a writhing snake of mass m_3 (Treat snake as rigid point particle. Treat P_1 who is clearly rigid at this point as a point.) At the right end is a person P_2 of mass m_2. At $t = 0$, P_1 throws the snake towards P_2 at a speed v. (i) What is V, the magnitude of the velocity of the boat and passengers relative to water when snake is airborne? (ii) How long does snake take to reach P_2? (iii) During this time how much has the boat moved to the left? (iv) Locate the CM at the end of all this snake throwing and show it is same as at beginning.

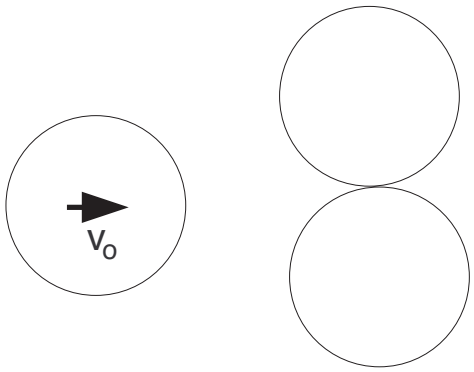

FIG. 1: The balls are identical and frictionless and the collision is symmetric, i.e., center of projectile is in line with point of contact of other two.

6. Find the CM of a cone of radius R and height h. (Think in terms of slices of thickness dy at height y.)

7. A person of mass $M = 32.5kg$ on ice disdainfully throws my quantum text book weighing $m = 2.25kg$ at $v_b = 12m/s$. The book is thrown from zero height and the total distance between the book and the offender is $15.2m$ when the book lands. At what angle was this excellent book thrown? How fast is the offender moving?

8. Block A of mass m is moving at velocity $+v$ towards mass B of mass $2m$ which is at rest. To its right and at rest is mass C of mass m. Find the ultimate velocities of all three masses assuming all collisions are elastic.

9. Two identical frictionless billiard balls are symmetrically hit by a third identical ball with velocity $\mathbf{i}v_0$ as in Figure (1). Find all subsequent velocities following this elastic collision. (Draw a picture at the moment of collision. What does "no friction" say about direction of forces?)