Physics 200
Practice Final
(Sketchy, as in outlined) Solution

I. (i) In time t the heater will have provided energy $Q_{\rm heater} = Pt$, where P is the power. We first compare this to the amount of energy needed to get the water to the boiling point (the symbols should be relativity self-explanatory; we will keep all the quantities positive here):

$$Q_{\rm boil} = (m_{\rm Al}c_{\rm Al} + m_{\rm water}c_{\rm water})\Delta T_{\rm boil},$$

which turns out to be larger than $Q_{\rm heater}$. So the water will not boil, and we can find the final temperature with

$$T = T_0 + \Delta T = T_0 + \frac{Q_{\rm heater}}{m_{\rm Al}c_{\rm Al} + m_{\rm water}c_{\rm water}}.$$

Let the the denominator be α for the next part.

(ii) It's clear this time that the water will not get to the boiling point. Let R be the rate of rain, so that the amount of rainfall is Rt, and let T' be the final temperature with the rain mixed in. Then

$$\alpha(T' - T) + (Rt)c_{\rm water}(T' - T_{\rm rain}) = 0,$$

which gives

$$T' = \frac{\alpha T + Rtc_{\rm water}T_{\rm rain}}{\alpha + Rtc_{\rm water}}.$$

II. Let m stand for the main pipe and v for the venturi. Then Bernoulli's equation tells us

$$\Delta P = \frac{1}{2}\rho(v_v^2 - v_m^2),$$

and by flow conservation we know $v_v = 4v_m$, so that

$$v_v^2 - v_m^2 = v_m^2\left[\left(\frac{v_v}{v_m}\right)^2 - 1\right] = 15v_m^2.$$

Therefore

$$\rho = \frac{2\Delta P}{v_v^2 - v_m^2} = \frac{2\Delta P}{15v_m^2}.$$

III. (i) From the adiabatic relation

$$P_B = P_A\left(\frac{V_A}{V_B}\right)^\gamma = \frac{P_A}{3^\gamma}.$$

(ii) Because the path CA is an isotherm, PV stays constant. therefore

$$P_C = P_A\frac{V_A}{V_C} = \frac{P_A}{3}.$$

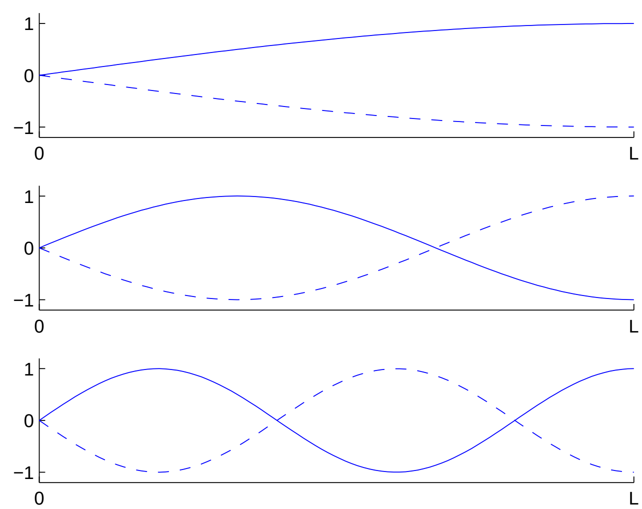
(iii) Clearly

$$W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1},$$
$$W_{BC} = 0,$$
$$W_{CA} = P_A V_A \ln \frac{V_A}{V_C},$$

so the total work done is $W = W_{AB} + W_{CA}$. Note that when you plug in numbers the total work done is negative. This means that that this cycle represents a refrigerator not an engine.

(iv) Since we complete an entire cycle, the net energy change is zero and the heat input is equal to the work done: $Q = W$.

IV. Since only one end is fixed, the allowed modes must fit an odd number of quarter-wavelengths in the length of the string. So the first three modes are $L = \lambda/4$, $3\lambda/4$, $5\lambda/4$:



The frequency is inversely proportional to wavelength, so the two higher frequencies are $3f_0$ and $5f_0$.

V. (i) Initially all the energy is stored as potential energy in the spring, so

$$E = \frac{1}{2}kx_i^2.$$

(ii) Using the conservation of energy

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \implies v = \sqrt{\frac{2E - kx^2}{m}}$$

where $x = 10cm$.

(iii) This means that all of the initial energy must be lost to friction, which does work $E_f = -\mu_k mg(x_f - x_i)$. Therefore

$$\mu_k = \frac{E}{mg(\Delta x)}$$

where $\Delta x = 15cm$.

VI. This is simply

$$v = \sqrt{\frac{\mu g y}{\mu}} = \sqrt{gy}.$$

VII. Please see the solutions to Problem Set 6, question 10. Note that the numbers are somewhat different here.

VIII. Please see the solutions to Problem Set 7, question 4.

IX. Please see the solutions to Problem Set 7, question 9.

X. Take the length of the vine to be ℓ and the distance across the gorge to be d. The total heigh, h Tarzan will be off the ground when he arrives at the other side of the gorge is

$$h = \ell - \sqrt{\ell^2 - d^2}.$$

To reach this height, we can use conservation of energy to see that he must start out with a minimum velocity given by

$$v = \sqrt{2gh}.$$

Tarzan and the rope are moving in a circle, so the tension is given by

$$T = m\frac{v^2}{\ell} + mg$$

where m is the mass of Tarzan.

XI. Angular momentum is conserved because the sum of the external torques is zero. This means that

$$I_i\omega_i = I_f\omega_f \implies \omega_f = \omega_i \frac{MR^2}{MR^2 + mr^2}$$

where M and R go with the inital mass and m and r correspond to the second smaller mass. The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, and similarly for the final kinetic energy. The fraction of kinetic energy lost is

$$1 - \frac{K_f}{K_i} = 1 - \left(\frac{\omega_f}{\omega_i}\right)^2 \frac{I_f}{I_i}.$$