

## Physics 200a PSII

1. Let  $\mathbf{A} = 3\mathbf{i} + 4\mathbf{j}$  and  $\mathbf{B} = 5\mathbf{i} - 6\mathbf{j}$ .
  - (i) Find  $\mathbf{A} + \mathbf{B}$ ,  $\mathbf{A} - \mathbf{B}$ ,  $2\mathbf{A} + 3\mathbf{B}$ , and  $\mathbf{C}$  such that  $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$ .
  - (ii) Find  $A$ , the length of  $\mathbf{A}$  and the angle it makes with the x-axis.
2. A train is moving with velocity  $\mathbf{v}_{TG} = 3\mathbf{i} + 4\mathbf{j}$  relative to the ground. A bullet is fired in the train with velocity  $\mathbf{v}_{BT} = 15\mathbf{i} - 6\mathbf{j}$  relative to the train. What is the bullets' velocity  $\mathbf{v}_{BG}$  relative to the ground?
3. Consider the primed axis rotated relative to the unprimed by an angle  $\phi$  in the counterclockwise direction.
  - (i) Derive the relation

$$A_x = A'_x \cos \phi - A'_y \sin \phi$$

$$A_y = A'_y \cos \phi + A'_x \sin \phi$$

that expresses unprimed components in terms of primed components of a vector  $\vec{A}$  using class notes if needed to get started.

- (ii) Invert these relations to express the primed components in terms of unprimed components. In doing this remember that the sines and cosines are constants and that we should treat  $A'_x$  and  $A'_y$  as unknowns written in terms of knowns  $A_x$  and  $A_y$ . (Thus multiply one equation by something, another by something else, add and subtract etc to isolate the unknowns. Use simple trig identities)
- (iii) Argue why one could have obtained this result easily by reversing  $\phi$ .

(iv) Consider a specific case  $A_x = 1$ ,  $A_y = 1$ ,  $\phi = \pi/4$ . What do you expect for  $A'_x$  and  $A'_y$  based on a sketch? Confirm by explicit calculation.

(v) Verify that the length squared of  $\vec{A}$  comes out same in both systems.

(Vi) Consider another vector  $\vec{B}$  and its components in the two frames. Show that

$$A_x B_x + A_y B_y = A'_x B'_x + A'_y B'_y.$$

We will understand this invariance later.

4. A particle is located at  $\mathbf{r}(t) = 14t\mathbf{i} + 6t^2\mathbf{j}$ . Find its position, velocity and acceleration at  $t = 2$  s.

5. At a wedding the 2m tall bride throws her bouquet with a velocity  $v_0 = 25\text{m/s}$  at an angle  $37^\circ$  above the horizontal. It is caught by a friend of height 1.5m. How long is the bouquet in flight and how far did it go horizontally? What was its maximum height above the ground?

6. Estimate the acceleration of the moon towards the earth given it orbits it once in 28 days at a radius of about a quarter of a million miles. (I know the units are funny and numbers are approximate. This problem tests your ability to give a quick and decent estimate, say to 10 percent .)

7. A jet pilot diving vertically down at 600 km/hr wants to make a quarter turn without experiencing an acceleration bigger than  $5g$ . At what height must the turn begin? Assume that the speed is constant and that after the quarter turn the plane, moving horizontally, is at ground level.

8. Here is problem designed so people in the life sciences will feel physics is relevant to them. A monkey is hanging from a height  $h$  and a person  $d$  meters away from the tree and on the ground, wants to zap it (in today's version with a tranquilizer gun and in the original version, a hunting rifle). He aims straight at the monkey and fires. This would of course work in the absence of gravity but show that it will work even in its presence provided the initial speed obeys  $v_0 > \sqrt{(d^2 + h^2)g/2h}$  What does this requirement ensure? Given

that this will also work if a pulse of laser light is used, what do you learn about light in a gravitational field?

People interested in other areas can replace monkey by suitable object, e.g., hard drive or a copy of Kant's Critique of Pure Reason.

9. Show that if a projectile is shot from a height  $h$  with speed  $v_0$  the maximum range obtains for launch angle  $\theta = \text{ArcTan} \left( \frac{v_0}{\sqrt{2gh + v_0^2}} \right)$ .